1 Financial Assets

A **financial asset** is a legal contract between two parties, the buyer and the seller. The contract works as follows:

i. The seller writes on a piece of paper a description of future promised payments. This piece of paper is the financial asset and whatever is written codifies the terms of the legal contract.

ii. The description can be as simple or as complex as desired. For example:

   - *"pay one dollar a day until the end of this year"*,
   - *"pay one dollar a day until the end of this year if, and only if, the stock market went up the previous day"*,
   - *"on the day Greece defaults on its debt, pay one dollar, and pay nothing otherwise"*.

iii. The buyer and the seller agree on a price P.

iv. The buyer gives the seller P dollars and the seller gives the piece of paper (the financial asset) to the buyer.

v. When one of the promised payments encoded on the paper comes due, the seller makes the payment to whomever happens to own the piece of paper at the time of the payment, be it the original buyer or a subsequent owner.

vi. If the seller does not make a payment as promised, the financial asset is said to be **in default**. Any violation of the promises, however small, triggers default. For example, default is triggered if payments are:

   - late by a single day, or
   - short by a single dollar.

   In case of default, the owner of the financial asset can take the seller to court to recover the value of the violated promises.

A **financial liability** is a contractual obligation to deliver the payments associated with a financial asset. In the situation considered above, the seller has a financial liability corresponding to the legal obligation to make the payments promised by the financial asset it created.

Because financial contracts might specify payments from the seller to the buyer, from the buyer to the seller, or both, it is not always the case that the seller ends up with a liability and the buyer with an asset. Indeed, many financial contracts structure future payments so that no money is exchanged when the contract is signed. In this case, neither buyer nor seller can be said to have an asset or a liability. Instead, we refer to the contractual obligations of the buyer and the seller as the two **legs of the contract**.

The terms and logistics surrounding financial assets can be arbitrarily complex. A single contract can stipulate payments among more than two parties, spell out what court jurisdiction can be used in the event of default, prohibit re-selling the asset to third parties, specify what accounts must be used for payments, what happens in case of default, and so on.

Payoffs can be cash payments in dollars or in other foreign currencies, deliveries of commodities such as gold, oil, or soybean, and even transfers of other financial assets. Therefore, when talking about financial assets, we often use the word **payoffs** rather than **payments** to highlight this generality, though both terms are largely interchangeable. When all payments are in dollars, they are usually referred to as **cash flows**.

Takeaway

- Financial assets are legal contracts defined by their payoffs.

1.1 Bonds

A bond is a financial asset. Like all other financial assets, it is a piece of paper with promises of future payoffs written on it. When a bond is created and sold for the first time, we say that the bond is **issued**.

What distinguishes bonds from other types of financial assets—what makes a bond a *bond*—is that the amount and timing of the promised payoffs are fixed at issuance. Absent default, the sizes and dates of the payoffs are thus known with certainty from the moment the bond is issued, and they never change throughout the bond's lifetime.

Zero-coupon bonds

The simplest kind of bond is one that promises a single payment at a specified date. Such a bond is called a **discount bond** or **zero-coupon bond**. Figure 1.1(a) provides an example. The promise is for $1,000, to be paid exactly one year after the bond is issued. The promised amount of $1,000 is called the **principal**, **face value**, or **par value** of the bond. The time left until the bond pays the principal is the **maturity** of the bond. After the bond is issued, the maturity decreases as time goes by. For example, if two months pass after a one-year-maturity bond is issued, the maturity of the bond is 10 months.

Coupon bonds

Figure 1.1(b) shows a different kind of bond, a **coupon bond**. It has the same face value and maturity as the bond in Figure 1.1(a), but includes two additional promises, known as **interest payments**, **coupon payments** or simply **coupons**. Coupons are promised payments that occur at fixed intervals. Coupons can be paid before or at maturity.

In the bond of Figure 1.1(b), the first coupon is paid six months after issuance, and the second one is paid on year after issuance, concurrent with the principal. Coupons are expressed as a percentage of face value; the ratio of coupon payments to the face value is the **coupon rate**. The coupon rate is always **annualized** or **per annum**. For the bond in Figure 1.1(b), the coupon rate is 2% to be paid semiannually (two times a year, every six months). Because the 2% rate is annualized, each semi-annual payment corresponds to only 1% of face value. With a face value of $1,000, each coupon payment is $10. Together, the payments of this bond are as follows:

- a coupon of $10 six months after issuance,
- a coupon of $10 plus the $1,000 principal (i.e., $1,010 total) one year after issuance.

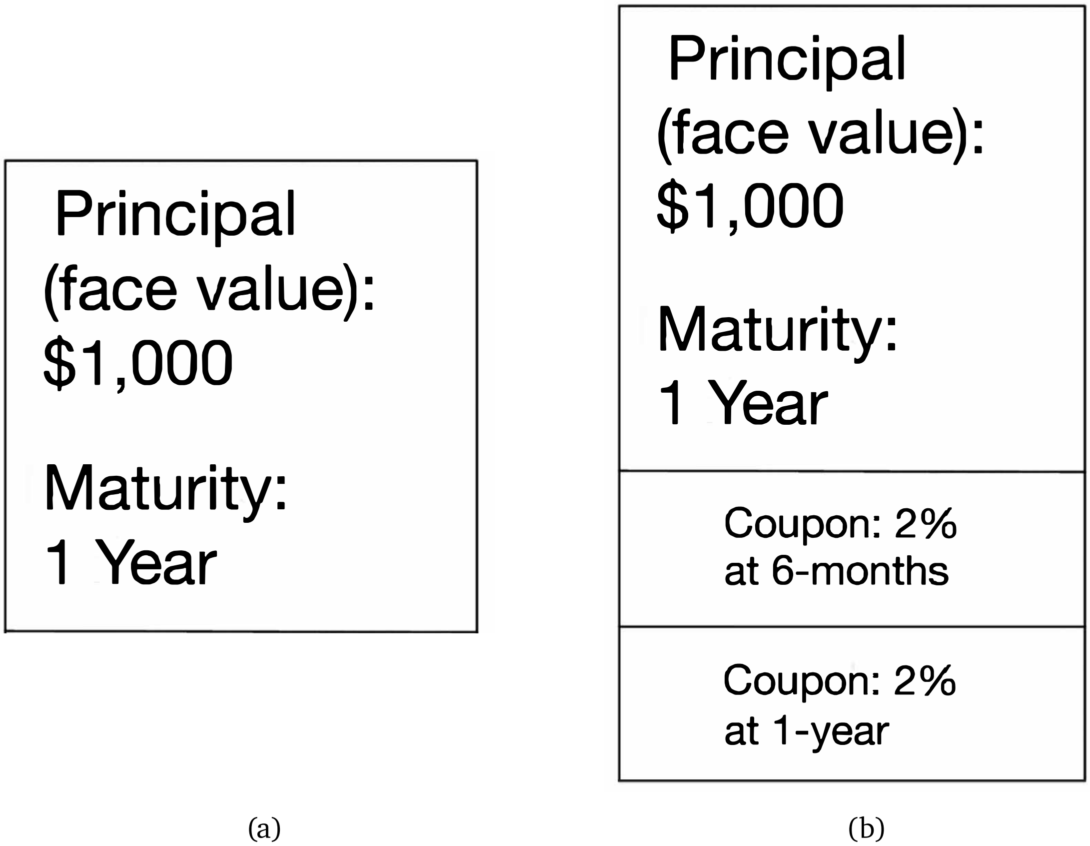

Figure 1.1: Panel (a) shows a one-year-maturity zero coupon bond with a principal of $1,000. This bond has a single fixed payment of $1,000 that occurs one year after the bond is issued.

Panel (b) shows a one-year-maturity coupon bond with a principal of $1,000 and a 2% coupon rate to be paid semiannually. This bond has two payoffs of fixed size at two fixed dates. The first payment occurs six months after the bond is issued and equals $10 (an annualized coupon rate of 2% is 1% over six months, and 1% of $1,000 is $10). The second payment occurs one year after the bond is issued and equals $1,010 (the principal of $1,000 plus the second coupon of $10).

Takeaway

- Bonds are financial assets with fixed payments.
- Zero-coupon bonds only pay at maturity.
- Coupon bonds also have intermediate payments.

Bond Prices

The date and amount of a bond's payment are written on the bond and do not change when the bond changes hands. The price of the bond—the amount that the buyer pays the seller to become the owner of the bond—is *not* written on the bond.

Instead, the price is determined by supply and demand. The price can be different each time the bond is traded. Supply and demand can take different forms depending on how markets are organized. For example, there can be a single seller and a single buyer that negotiate over the price, many traders who buy and sell at a price that adjusts continuously to clear the market, or a single seller and many buyers with the price determined by an auction[1]

Bond Yields

The **interest rate** of a zero-coupon bond is the bond's rate of return until maturity. Denote the price of the bond at time t by B_t. Then the bond's interest rate at time t, denoted by R_t, is

$$R_t = \frac{\$1,000 - B_t}{B_t}. \tag{1.1}$$

The **yield to maturity** on a zero-coupon bond is the same as its interest rate; we use both terms interchangeably. Yield to maturity is often shortened to **yield**.[2]

Equation (1.1) can be solved for the bond's price B_t to get

$$B_t = \frac{\$1,000}{1 + R_t}. \tag{1.2}$$

Equations (1.1) and (1.2) allow us to convert between the bond's price B_t and yield R_t: prices and yields convey the same information. A higher price is associated with a lower yield, and a lower price with a higher yield. When we talk about bond yields, we are

[1] U.S. government bonds, for example, are typically sold at issuance through an auction. You can participate in these auctions yourself and buy U.S. government bonds directly from the U.S. Treasury at Treasury Direct.

[2] A bond's **current yield** is the ratio of the coupon payment to the current price of the bond. The current yield is a different concept from yield to maturity. If you see the word yield by itself, it can mean current yield or yield to maturity depending on the context.

therefore just talking about the bond's price.

Bond yields are always expressed in annualized terms. To annualize a yield, we divide the raw (non-annualized) interest rate R_t by the maturity of the bond, with maturity expressed in years. Denote the maturity of the bond at time t expressed in years by m_t. Then the annualized yield is[3]

$$\text{Annualized } R_t = \frac{R_t}{m_t}.$$

For coupon bonds, the **yield to maturity** is the bond's return if held to maturity, assuming all coupon and principal payments are made as scheduled.

Takeaway

- Bond prices and yields convey the same information.
- An increase in the price of a bond is equivalent to a decrease in the yield of the bond (and vice-versa).

Risk-free bonds

Bonds that never default are called **risk-free** or **riskless**. Of course, no bond is truly default-free. However, U.S. government bonds are generally considered very close to default-free, so treating them as riskless is a reasonable approximation in many economic applications.

The **risk-free rate** is the yield of a riskless zero-coupon bond.

[3]This definition assumes interest compounds continuously. An alternative choice is to compound period by period, in which case the annualized yield $R_t^{\text{annualized}}$ is defined by

$$(1 + R_t^{\text{annualized}})^{m_t} = (1 + R_t)\,.$$

It is always possible to translate interest rates that are annualized using continuous compounding to interest rates that are annualized by using period-by-period compounding. Both measure the same economic quantity in different units. Annualized yields using continuous and period-by-period compounding are numerically close to each other—and hence essentially interchangeable without converting units—when $R_t^{\text{annualized}}$ is not too big (say, 10%-15% or less). In these notes, we use continuous compounding because calculations are easier.

Price Risk

Let R_{mt} be the annualized yield at time t for a zero-coupon bond with maturity m. At time t, R_{mt} is known. After all, knowing R_{mt} is the same as knowing the bond's price and, if we can buy the bond, then its price must be known!

In contrast, the future yield $R_{m,t+1}$ is *not* known at t; it is first known at time $t+1$. However, at time t we can form some expectation—a best guess—regarding what $R_{m,t+1}$ will be. We denote this **expected yield** or **expected interest rate** by $R^e_{m,t+1}$. The expected yield $R^e_{m,t+1}$ *is* known at t.

Investing $1,000 in a riskless two-year-maturity zero coupon bond at time t has payoffs of $0 at $t+1$ and

$$\$1000 \times (1 + 2R_{2t}) \tag{1.3}$$

at $t+2$. The number 2 in front of R_{2t} is there because R_{2t} is expressed in annualized terms.

Investing $1,000 in a riskless *one*-year-maturity zero coupon bond at time t has a payoff equal to

$$\text{payoff}_{t+1} = \$1000 \times (1 + R_{1t})$$

at $t+1$. Reinvesting payoff_{t+1} at $t+1$ on new riskless one-year-maturity zero coupon bond (that matures at time $t+2$) results in a payoff equal to

$$\begin{aligned} \text{payoff}_{t+2} &= \text{payoff}_{t+1} \times (1 + R_{1,t+1}) \\ &= \$1000 \times (1 + R_{1t}) \times (1 + R_{1,t+1}) \end{aligned} \tag{1.4}$$

at $t+2$. The *expected* payoff at $t+2$ is the same as the actual payoff in Equation (1.4) but replacing the actual or **realized yield** $R_{1,t+1}$ by the expected yield $R^e_{1,t+1}$.

Reinvesting the principal payoff of a bond into a new bond of the same maturity is called **rolling over** the bond. Equation (1.4) gives the payoff of rolling over a one-year-maturity bond for two years.

Buying the two-year bond at time t entails no risk: the payoff in Equation (1.3) is known at t. In contrast, rolling over a one-year bond for two years has a risky payoff because the future yield $R_{1,t+1}$ is not known at t. This kind of risk is called **price risk**. It is different from default risk, which is the risk that bond payments are not honored. Confusingly, risk-free bonds can have price risk.[4]

Takeaway

- There are two kinds of risks for bonds: default risk and price risk.
- Default risk is the risk that payments are not honored.
- Price risk is the risk that bond yields change in the future.

Bond Risk Premia

Equilibrium in Bond Markets Determines Risk Premia Investors do not like risk. If buying the two-year bond and rolling over the one-year bond twice had the same expected payoffs, investors would always prefer to buy the two-year bond. No one would want to hold the existing supply of one-year-maturity bonds, preventing the market for one-year bonds from being in equilibrium.

For the bond market to be in equilibrium, rolling over the one-year bond twice must have a higher expected payoff than buying the two-year bond. The difference in expected payoffs between the two options is a **risk premium**. The risk premium is the compensation for taking on the price risk associated with rolling over the one-year bond. In the context of the yield curve, the risk premium is also referred to as the **term premium**.

Mathematically, equilibrium in the bond market requires that

$$\$1,000 \times (1 + 2R_{2t}) = \$1,000 \times (1 + R_{1t})(1 + R^e_{1,t+1}) + x_2. \tag{1.5}$$

The left-hand side of Equation (1.5) is the payoff from investing in the two-year bond, taken from Equation (1.3). The right-hand side is the expected payoff from rolling over the one-year bond twice (Equation (1.4) with $R_{m,t+1}$ replaced by $R^e_{m,t+1}$), plus the risk premium x_2.

Equation (1.5) can be simplified by ignoring a small cross-term. The product on the right-

[4] Economics: Why make it easy when we can make it harder? Don't get me started about *heteroskedasticity*.

hand side can be expanded as:

$$(1 + R_{1t})(1 + R^e_{1,t+1}) = 1 + R_{1t} + R^e_{1,t+1} + R_{1t}R^e_{1,t+1}.$$

If R_{1t} and $R^e_{1,t+1}$ are not too big (say, 10% or less), then the term $R_{1t}R^e_{1,t+1}$ is much smaller than the other terms. After canceling the \$1,000 from both sides, Equation (1.5) can thus be approximated by ignoring the small term $R_{1t}R^e_{1,t+1}$, which results in

$$1 + 2R_{2t} = 1 + R_{1t} + R^e_{1,t+1} + x_2,$$

or

$$R_{2t} = \frac{1}{2}(R_{1t} + R^e_{1,t+1} + x_2). \tag{1.6}$$

In words, Equation (1.6) says that the two-year yield is the average of the future path of one-year yields, plus a risk premium.

Comparing an m-year bond to rolling over one-year-maturity bonds for m periods gives the general formula:

$$R_{mt} = \frac{1}{m}(R_{1t} + R^e_{1,t+1} + ... + R^e_{1,t+m-1} + x_m). \tag{1.7}$$

Once again, the m-year yield is the average of the future expected one-year yields, plus a risk premium.

Takeaway

- Equilibrium requires that the yield of a long-maturity bond equals the average of future expected one-year yields, plus a risk premium.

Nominal and Real Interest Rates

The interest rates and yields we have considered thus far are all **nominal** because the bond payments are denominated in dollars.

The **real interest rate**, also called the **real yield**, is defined as the nominal interest rate minus expected inflation:

$$r_{mt} = R_{mt} - \pi^e_{mt}, \tag{1.8}$$

where r_{mt} is the real interest rate (or real yield), R_{mt} is the nominal interest rate (or nominal yield), and π^e_{mt} is the inflation rate expected to prevail between time t and time $t + m$.

The real yield is the yield on a **real bond**, which has payments specified not in terms of dollars but in terms of the representative basket of goods and services of the economy. Investing an amount equal to one basket of goods in zero-coupon real bonds with maturity m at time t has a promised payoff of $(1 + r_{mt})$ baskets of goods at $t + m$.

Combining the definition of the real yield in Equation (1.8) with Equation (1.7) gives:

$$\begin{aligned} R_{mt} &= \frac{1}{m}\Big[(r_{1t} + \pi^e_{1t}) + (r^e_{1,t+1} + \pi^e_{1,t+1}) + ... + (r^e_{1,t+m-1} + \pi^e_{1,t+m-1}) + x_m\Big] \\ &= \frac{1}{m}\Big[(r_{1t} + r^e_{1,t+1} + ... + r^e_{1,t+m-1}) + (\pi^e_{1t} + \pi^e_{1,t+1} + ... + \pi^e_{1,t+m-1}) + x_m\Big]. \end{aligned}$$

This last equation expresses the nominal m-year yield as the average of the expected one-year *real* yields, plus the average expected inflation, plus the risk premium. It is a useful decomposition of long-term nominal yields into three parts: one from real yields, another from inflation, and a third from risk premia.

1.2 Stocks

Stocks are financial assets. Like all other financial assets, they are defined by the future payoffs they promise.

Stocks are **issued**—created and first sold—by corporations, which we refer to as firms for short. A stock promises its holder payoffs equal to a fixed fraction of the lifetime profits of the firm. The size of the fraction is determined by the total number of stocks issued. For example, if a firm has issued 100 stocks, each stock promises to pay out 1% of the firm's lifetime profits. The payments that stock owners receive are called **dividends**.

Legally, each stock represents the ownership of a fraction of the firm. Because stocks represent shares of ownership, they are also called **shares** and the people who own them are **shareholders**.

The promise to pay a share of *lifetime* profits does not mean that all profits must be paid out to shareholders immediately after profits are made. Indeed, even though all profits will ultimately be paid as dividends, stocks make no promises about when dividends will be paid or how much will be paid each time. Dividend decisions are made by each firm's board of directors, who evaluate whether reinvesting profits or distributing them as dividends will generate greater value for shareholders.

Start-ups and fast-growing firms don't usually pay dividends. Many large, established companies pay dividends every quarter, typically a fixed percentage of the profits earned since the last dividend. Google, founded in 1998, did not pay dividends (through its parent company, Alphabet) until 2024.

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